Wednesday, 2008 September 17, 00:42:38
JAW, Monday, Tuesday, 2008 September 16, 23:10:43

Transform from Ω to Another Basis *C*

We know that alternate sets of color matching functions can exist that are linear transformations of one another. One may ask, given two such sets of functions, how to find the 3×3 matrix that converts the one to the other? The discussion below applies if one of the sets of CMFs is the orthonormal basis.

Suppose that Ω is the orthonormal basis, derived from the 2° observer. We recall then that Ω comprises 3 column vectors,

$$\Omega = [|\omega_1\rangle \; |\omega_2\rangle \; |\omega_3\rangle] \quad . \tag{1}$$

Suppose further that *A* is a different basis, also derived from the 2° observer. It's not important that the specific standard observer is used, but in any case, we know that the one set of functions is a linear transformation of the other set, which can be written as

$$A = \Omega T \quad . \tag{2}$$

Since each basis is represented by the 3 columns of a matrix, the transformation is written as square matrix *T*, which then post-multiplies Ω to give *A*.

The orthonormal property of Ω can be written in matrix form:

$$\Omega^{\mathrm{T}}\Omega = \mathrm{I}_{3\times 3} \quad . \tag{3}$$

Of course, superscript T indicates matrix transpose, nothing to do with square matrix *T*.

We now wish to solve Eq. (2) for *T* . Left-multiply Eq. (2) by Ω^{T} to obtain

$$\Omega^{\mathrm{T}} A = \Omega^{\mathrm{T}}\Omega T \quad . \tag{4}$$

Note from Eq. (3) that $\Omega^{\mathrm{T}}\Omega$ is the identity matrix, so it can be omitted in Eq. (4) and we find

$$T = \Omega^{\mathrm{T}} A \quad , \tag{5}$$

the result that was sought.

Eq. (5) has practical application in computer work. It can happen that Ω and some *A* have been found from a starting point such as [x-bar y-bar z-bar] by some steps, but *T* is not known.

The derivation could jump from Eq. (2) to Eq. (5) with a few words of explanation, but here the assumptions and details have been explained.

Jim Worthey home page: http://www.jimworthey.com .