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Two Applications of Vectorial Color: Camera Design and Lighting of Colored Objects

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Spoken Presentation Itself

<OSA's projector has pixel dimensions1024 wide x 768 high. The image science-timeline.png is 900 wide x 505 high. That leaves **263** vertical pixels for a few formulas or whatever. Cornsweet_Cover.png is **245** px high.>

Lighting is a peculiar area of study. To see it in relation to other topics, let's review the history of science, in about 5 minutes. I made a chronology in the form of a long web page, which you can review on my web site. From that version, Nick Worthey developed this graphical timeline.

Books now teach that momentum *mv* is conserved in a collision, but kinetic energy $\frac{1}{2}mv^2$ can also be conserved. These confusing facts were resolved during the "*vis viva dispute*," described in a *Physics Today* article. There was a 100-year age of *vis viva* within the 200-year era of mechanics. Science confronts a tricky situation like energy and momentum, and **organizes the facts**.

From Galvani's twitching frog legs to Edison's light bulb patent, systematic study of electricity took about 100 years. From Newton's "New Theory about Light and Colors" to Frederic Ives's color photos based on the trichromatic theory of color vision was about a 200-year era of color fundamentals.

In the 1800s, great minds worked to complete classical physics. For example, Newton applied $F=ma$ to the motion of planets, but he didn't have vector methods. Hamilton invented quaternions, which have important uses, but around 1900 Gibbs and others pieced together modern vector algebra. Boltzmann and Gibbs advanced thermodynamics. Modern light measurements were launched when Langley's bolometer and Grayson's diffraction gratings emerged in about 1900.

Physicists rushed to finish an old era and make way for the new. Einstein's four articles in 1905 turned physics toward quantum mechanics and relativity. In the larger world, Edison's light bulb was patented in 1880 and in 1889 German-educated mathematician C. P. Steinmetz moved to the USA and found work in the electrical industry. Beyond his achievements as an inventor, he stands out as one who literally wrote the book for 20th century electrical engineering. He improved the methods for AC circuit calculations, then wrote books on AC phenomena and other electrical subjects. As the gaps of classical physics were filled in, a transition occurred, shown in Fig.1 as The Big Bang of Engineering. Authors such as Steinmetz found among the great ideas a portion that could be immediately applied, creating workaday methods grounded in physics.

Electrical engineering began with a great legacy, including physics and math, but lighting did not. For lighting there was no legacy from Newton or Maxwell, and no Steinmetz to put lighting science into practice. There is in fact a lighting book by Steinmetz, based on lectures that he gave. He covered physics and light measurement, but in speaking to the needs of the visual system, he concluded "Other physiological requirements are still very little understood or entirely unknown."

In the timeline, we made Claude Shannon a lighting expert, but that's wishful thinking. If he had thought about lighting after creating the theory of information in 1948, then he might have noticed that the eye is an information channel, dependent on lighting to create highlights, shading, colors and other cues. Notice at the lower right: the analytical methods for lighting and applied color are still under development.

"A Mixture of Monochromatic Yellow and Blue Light"

The Illuminating Engineering Society, the IES, was founded in 1906. In 1912, in Volume **7** of *Transactions of the IES*, Herbert Ives published this passing remark:

"It is, for instance, easily possible to make a subjective white, as by a mixture of monochromatic yellow and blue light. A white surface under this would look as it does under 'daylight' but hardly a single other color would." -- H. E. Ives

The black lines are Ives's original drawing, the orange and blue lines are added to represent his "mixture of monochromatic yellow and blue light." Color vision is trichromatic, but monochromatic yellow stimulates two receptor systems because of the large overlap of the red and green sensitivities.

Ives Took a Scientific Approach.

Ives's comment stands out for its scientific insight:

- He gave a simple example based on the facts of color vision.
- He anticipated issues that arose later with commercial lights.

2-bands Issue Since 1912

Bill Thornton re-discovered the 2-bands issue and published the idea in 1978. I took the idea from Thornton and from Ives and used it over time. But the usual lighting discussions still ignore Ives's insight from a century ago.

Why Is it Hard to Discuss Vision and Lighting?

Vision by 2 eyes has been developing for a long time. The normal process of seeing is effortless. Its complexity is hidden. It is work to **stop** taking vision for granted and talk about a detail such as the role of the light.

Vectorial Color Overview (part 1)

Let's do a quick overview of vectorial color. Narrow-band lights of unit power but different wavelengths plot to vectors that differ in amplitude and direction. Colored lights then add vectorially. In the second figure, the arrows in the curving chain add to make the so-called Equal Energy Light.

Vectorial Color Overview (part 2)

The three functions at left are the orthonormal opponent color-matching functions. In short, the orthonormal basis. For convenience the functions [**little**] ω_1, ω_2, ω_3 become the columns of matrix Ω [**big Omega**]. Then matrix algebra can be used to find big V, a 3-vector. This is normal colorimetry, slightly revised.

Vectorial Color Overview (part 3)

Vectorial color was **not** invented to solve one problem. It emerges from fundamental research. Now I'll fill in more background.

Legacy Understanding

Data from a color-matching experiment look like the first graph. The vertical scaling is not arbitrary. The data are called color-matching functions; that name can also be used for other sets of related functions. Cone sensitivities are linear combinations of the original data. So are the arbitrary functions of the XYZ system, or the opponent color functions, the 4th graph.

3 functions become the columns of a matrix:

Any set of 3 functions can become a matrix of 3 long columns.

SPD as a Column Vector

Now if a light's SPD, for example any of the graphs on the right, is written as a column vector, the product $A^{\mathrm{T}} L$ is a little 3-vector, the tristimulus vector. That vector could be the usual big X, big Y, big Z, but there is a better way.

Color Mixing Experiments: Choice of 3 λs

A book may tell you that two scientists, Wright and Guild, used different primaries, and you can calculate the effect of changing the primary set. This first animated graph shows that different primaries led to different data. **<Scroll to second animation.>**

The second graph illustrates a discovery of Thornton. As one primary changes, the peaks of the color-matching data tend to stay at the same wavelengths. But the choice of primaries affects the optical power that's needed to match the test light. The least-power primaries are Thornton's "prime colors."

Strongly Acting Wavelengths

In the 1970s, developing a new idea, Thornton found 3 **strongly acting wavelengths**. Later he saw the strongly acting wavelengths at work in the original color matching data.

Some Credit to Tom N. Cornsweet

In his 1970 book, Tom Cornsweet used vector diagrams to discuss overlapping receptor sensitivities.

Jozef Cohen and the Fundamental Metamer

< Dust jacket and photo briefly, then scroll to text etc. >

Cohen sought an **invariant presentation** of color mixing facts.

- Consider any light $L(\lambda)$.
- $L^*(\lambda)$ = the Fundamental Metamer of $L(\lambda)$ = the projection of $L(\lambda)$ into the space of color-matching functions, CMFs.
- Start with $L(\lambda)$, and with a set of 3 CMFs. Find the linear combination of the CMFs that is a least-squares best fit to $L(\lambda)$. That is the projection. That's the fundamental metamer.
- $L^*(\lambda)$ is invariant. For example, start with any of the 4 sets of CMFs at the right. $L^*(\lambda)$ comes out the same.
- Cohen found an easy method to obtain $L^*(\lambda)$.
- Cohen's method for finding L^* led him to other ideas.

Examples of Fundamental Metamers.

In these 5 examples, the two functions **are metamers**. The light and its fundamental metamer would match to the 2-degree observer. In each case, the smoother function is the fundamental metamer of the other one.

Joe Cohen's easy method: Matrix *R*

As before, let A be a matrix whose columns are a set of CMFs. Given L and A, we want to find L^*, the fundamental metamer. Now you might look up Moore-Penrose pseudo-inverse and get a numerical answer with that. Lucky for us, Cohen did not have Wikipedia and solved the math himself, which led him to more ideas.

Long story short, Cohen's method:

$$L^* = R\,L \quad , \qquad (2)$$

where

$$R = A(A^{\mathrm{T}}A)^{-1}A^{\mathrm{T}} \quad . \qquad (3)$$

Matrix *R*, continued.

Fun Facts about Matrix R

- Cohen often used "Matrix R" as the name for his research topic.
- Matrix R is a large square array. If the domain of λ has 471 steps, then R is a 471 x 471 matrix. That causes no trouble, but don't try to print it.
- **Matrix R itself is invariant.** Sets of CMFs that are different (but equivalent) lead to the same big array R.
- Matrix R is a projection matrix. It is symmetrical.
- The columns (or rows) of R are the fundamental metamers of the spectrum.
- In other words, the columns of R, interpreted as 3-vectors, trace the Locus of Unit Monochromats, the LUM.
- It is easier to explain and draw the LUM using the orthonormal basis.
- Some proofs about R are on my web site, as well as in Cohen's articles and book.

Fundamental Metamer Example.

The graph shows D65 and its fundamental metamer. The two curves are metamers in the ordinary sense. The purple curve is found by $L^* = RL$, where the formula for *R* was given above.

Orthonormal Opponent Color Matching Functions

In the past—since 1980—I had used opponent color algebra. In 2003, I discovered that a set of Orthonormal Opponent Color Matching Functions is closely related to Cohen's Matrix R, fundamental metamers, and the vector diagrams that he made.

Orthonormal Opponent Color Matching Functions: Description.

Little omega-1 is the achromatic function, proportional to the usual y-bar. It is a weighted sum of red and green cone sensitivities, with no blue input.

Little omega-2 is the red-green opponent function, orthogonal to omega-1, with no blue input.

Omega sub 3 is the blue-yellow function, and uses all 3 cone functions.

The omegas are orthogonal and normalized.

Bra and Ket Notation

I use bra and ket notation to distinguish column vectors from row vectors, and to indicate inner products.

Working Class Summary

Many clever ideas are merged in the orthonormal basis, but in the end it is not radically new.

1. ω_1 is proportional to old-fashioned y-bar (not shown). So ω_1 is not new.
2. ω_3 is similar to the old z-bar, which is essentially a blue cone function.
3. In the XYZ system, x-bar is an arbitrary magenta primary. It is replaced in the new system by ω_2 , a red-green opponent function. The opponent function confronts the overlap of cone sensitivities by finding a difference of red and green.

Let me say that again. An important feature of human vision is the overlapping sensitivities of the red and green cones. An opponent system **confronts the issue of overlap**. It re-mixes the red and green signals into ω_1 = red + green, and ω_2 = red - green.

Fun with Matrices

in short here, $$\Omega^T\Omega = I_{3\times3} \quad . \qquad (15)$$

If you multiply Ω and its transpose in the reverse order, you get a different interesting result:

$$\Omega\Omega^T = R \quad . \qquad (16)$$

Convenience of the Orthonormal Basis

To repeat,

$$\Omega^T\Omega = I_{3\times3}$$

and

$$\Omega\Omega^T = R \quad .$$

Recall from a few screens above:

This is just a reminder, how matrix big Omega is used to find V, the tristimulus vector.

Meaning of Tristimulus Values

v_1 = whiteness = achromatic or black-white dimension

v_2 = **red**ness or **green**ness

v_3 = **blue**ness or **yellow**ness

So, we can say that the **tristimulus values** have **intuitive meaning**.
But that's not all.

$$|L^*\rangle = v_1|\omega_1\rangle + v_2|\omega_2\rangle + v_3|\omega_3\rangle$$

If we express the fundamental metamer L^* by an orthonormal function expansion, the tristimulus values are the coefficients. The **same values** have **mathematical meaning**.

Graphing a Vector

So, we can calculate one vector from the origin.

5 Vectors

Or five vectors from the origin.

5 Vectors Added

Or add some vectors **vectorially**. That is they add tail to head.

Locus of Unit Monochromats

The vectors of the locus of unit monochromats are the rows of big Omega.

Locus of Unit Monochromats (continued)

If the LUM is shown as a surface, the locus is really the curve along the edge.

Wavelengths of Strong Action

For practical purposes, the wavelengths of strong action are the ones that give the longest vectors, within a nanometer or two.

Composition of a White Light

A simple white light is the so-called equal energy light, shown as the black line, or the green dots.

Composition of a White Light (continued)

Find the vector for each little band of the spectrum, then add them tail to head. The **red** and **green** components of a white light cancel each other out. They need to be present in order to reveal red and green objects. **This is the issue from Herbert Ives in 1912.**

< Click Skip one item **>**

Some Lights Have Less Red and Less Green

The same white point can be reached by different lights in different ways. SPDs of two lights are plotted on the right. They are color matched, but one is daylight and one is High Pressure Mercury Vapor.

Comparing Color Composition of Lights

The mercury light radiates most of its power in a few narrow bands, leading to a few long arrows that leap toward the final white point. Compared to “natural daylight,” the mercury light makes a smaller swing towards green, and a smaller swing back towards red. Such a light would leave the red pepper starved for red light with which to express its redness.

<Scroll down>

You can see the same thing when the vectors are projected into the v1 – v2 plane.

<Scroll down>

Here the v2 – v3 plane is used to show how some colored papers lose their redness and greenness under the mercury light.

Cameras and the "Maxwell-Ives Criterion"

Frederic Ives, the father of Herbert Ives, applied an idea from Maxwell:

“A color camera should have the same sensitivities as the eye.”

A modern statement:

“For color fidelity, a camera’s spectral sensitivities must be linear combinations of those for the eye."

The eye has an invariant LUM, and so does the camera.

There is **One Stumbling Block**

To compare the eye's LUM with the camera's, we need a way to put them in good alignment.

Compare Camera LUM to that of Human

Please look at the big picture here. The spheres trace the LUM of a certain Dalsa sensor. To the extent that the LUMs are the same, the sensor meets the Maxwell-Ives criterion. The arrowheads trace the best fit of camera functions to human.

The Fit First Method

The trick of the Fit First Method is to find the best fit first, then find the LUM from that.

The computer code has 3 steps.

The first step finds matrix *R* for the camera. The second step finds the 3 best-fit functions, fitting the camera to the eye. The third step orthonormalizes those functions to find the camera's LUM.

<Scroll on down to simple graph.>

In this simple graph, the camera is a Nikon D1. The dashed curve is omega sub one for human. The solid green line is a linear combination of the camera's sensitivities, a best fit to the dashed line. One matrix multiplication does 3 of these best fits.

Step **3**, Orthonormalize the Re-mixed Camera Functions.

Since the re-mixed camera functions are computed separately, they are not orthonormal and would not combine to map out a true Locus of Unit Monochromats. But they mimic **Ω** and are in the right sequence. We need to make them orthonormal, which is what the Gram-Schmidt method does, Step **3**.

<Just show the graphs, don't chat about them.>

Camera Example, Nikon D1

OK, here are the camera sensitivities for that Nikon D1.

< Scroll down. >

Here are 2 views of the comparison of camera and eye. You can see that there’s a good fit in some parts of the spectrum, not in others.

< Scroll on down until “Skip to Joe Photo.” >

There are more examples on my web site. There’s a link right here.

< Now, **Skip to Joe Photo.** >

This shows me, Joe Cohen, and my son Nick 21 years ago. **Thank you.**